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Productions of X(1835) as baryonium with sizable gluon content

Gui-Jun Ding[1,a], Rong-Gang Ping[2], and Mu-Lin Yan[1]

[1] Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, PRC
[2] Institute of High Energy Physics, P.O. Box 918(4), Beijing 100049, PRC

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Abstract. $X(1835)$ has been treated as a baryonium with sizable gluon content, and to be almost flavor singlet. This picture allows us to rationally understand $X(1835)$ production in J/ψ radiative decays, and its large couplings with $p\overline{p}$, $\eta'\pi\pi$. The processes $\Upsilon(1S) \to \gamma X(1835)$ and $J/\psi \to \omega X(1835)$ have been examined. It has been found that $Br(\Upsilon(1S) \to \gamma X(1835))Br(X(1835) \to p\overline{p}) < 6.45\times10^{-7}$, which is compatible with CLEO's recent experimental result (Phys. Rev. D **73**, 032001 (2006) hep-ex/0510015). The branching fractions $Br(J/\psi \to \omega X(1835))$, $Br(J/\psi \to \rho X(1835))$ with $X(1835) \to p\overline{p}$ and $X(1835) \to \eta'\pi^+\pi^-$ have been estimated by the quark-pair creation model. We show that they are heavily suppressed, so the signal of $X(1835)$ is very difficult, if not impossible, to be observed in these processes. The experimental checks for these estimations are expected. The existence of the baryonium nonet is conjectured, and a model-independent derivation of their production branching fractions is presented.

PACS. 12.38.-t Quantum chromodynamics – 12.38.Qk Experimental tests – 12.39.Mk Glueball and nonstandard multi-quark/gluon states – 12.39.St Factorization

1 Introduction

Recently, the BES Collaboration has observed a new resonant state $X(1835)$ in the $\eta'\pi\pi$ invariant mass spectrum in the process $J/\psi \to \gamma\pi^+\pi^-\eta'$ [1] with a statistical significance of 7.7σ. The fit with the Breit-Wigner function yields mass $M = 1833.7 \pm 6.1(\text{stat.}) \pm 2.7(\text{syst.})\,\text{MeV}/c^2$, width $\Gamma = 67.7 \pm 20.3(\text{stat.}) \pm 7.7(\text{syst.})\,\text{MeV}/c^2$ and the product branching fraction $Br(J/\psi \to \gamma X(1835))Br(X(1835) \to \pi^+\pi^-\eta') = (2.2 \pm 0.4(\text{stat.}) \pm 0.4(\text{syst.})) \times 10^{-4}$. A narrow near-threshold enhancement in the proton-antiproton ($p\overline{p}$) mass spectrum was observed from the radiative decay $J/\psi \to \gamma p\overline{p}$ [2]. This enhancement can be fitted with either an S- or P-wave Breit-Wigner resonance function. In the case of S-wave fit, the peak mass is $M = 1859^{+3}_{-10}(\text{stat.})^{+5}_{-25}(\text{syst.})\,\text{MeV}/c^2$ with total width $\Gamma < 30\,\text{MeV}/c^2$ at 90% confidental level and the product branching fraction $Br(J/\psi \to \gamma X)Br(X \to p\overline{p}) = (7.0 \pm 0.4^{+1.9}_{-0.8}) \times 10^{-5}$.

The masses of the two structures observed in both $J/\psi \to \gamma p\overline{p}$ and $J/\psi \to \gamma\eta'\pi^+\pi^-$ channels are overlapped and 0^{-+} quantum number for the resonance in $\eta'\pi^+\pi^-$ channel is possible. A question arise if they are the same state, in ref. [1] an argument is presented, if the final-state interaction is included in the fit of the $p\overline{p}$ mass spectrum, the width of the resonance observed in $\gamma p\overline{p}$ channel will become larger. Therefore, the X observed in both $p\overline{p}$ and $\eta'\pi^+\pi^-$ channels could be the same state and it is named as $X(1835)$ in ref. [1]. And this state couples strongly with $p\overline{p}$ and $\eta'\pi^+\pi^-$; in the recent talk of BES [3], the estimation of $Br(J/\psi \to \gamma X(1835)) \sim (0.5\text{–}2) \times 10^{-3}$, $Br(X \to p\overline{p}) \sim (4\text{–}14)\%$ is presented.

However, recently, a negative experimental result has been reported by the CLEO Collaboration [4]. They claimed that in the radiative decay of $\Upsilon(1S)$ the narrow enhancement observed by BES near the $p\overline{p}$ mass threshold is not seen. The upper limit of the product branching fraction for the decay $\Upsilon(1s) \to \gamma X(1835)$, $X(1835) \to \gamma p\overline{p}$ has been determined to be $Br(\Upsilon(1S) \to \gamma X(1835))Br(X(1835) \to p\overline{p}) < 5 \times 10^{-7}$ [4].

Moreover, we would like to mention another problem, because $Br(J/\psi \to \gamma X(1835)) \sim (0.5\text{–}2) \times 10^{-3}$, claimed by BES in [1], is rather larger among J/ψ decays and ω is a photon-like vector meson with negative G parity; an experimental measurement of $Br(J/\psi \to \omega X(1835))$ seems to be practicable in BES, or at least the signal of $J/\psi \to \omega X(1835)$ should be seen in BES. However, there are still not yet any results on this matter reported by BES, therefore it is urgent to discuss the problem that whether the fact that the signal of $J/\psi \to \omega X(1835)$ is not revealed at the present stage contradicts the existence of $X(1835)$ or not.

[a] e-mail: dingjj@mail.ustc.edu.cn

In this case, the existence of $X(1835)$ seems to become a puzzle. Therefore, it is worth pursuing both the reasons why $Br(\Upsilon(1S) \to \gamma X(1835))$ is so small that $Br(\Upsilon(1S) \to \gamma X(1835))Br(X(1835) \to p\overline{p}) < 5 \times 10^{-7}$ and the reasons why there is still not yet any information on $J/\psi \to \omega X(1835)$ reported by BES. In this work we try to answer the above questions, and try to illustrate that the absence of the $X(1835)$ signal from the two processes at the present stage is due to the special structure of $X(1835)$.

The theoretical interpretation of this exotic state is a great challenge, and many proposals have been suggested [5–15]. Some of them interpret $X(1835)$ as a $p\overline{p}$ bound state [6,9–11], and large enough binding energy to bind proton and antiproton together has been derived from the constitute quark models [11]. On the other hand, some authors identify $X(1835)$ as a pseudoscalar glueball [12,14], and in refs. [10,13] the authors claim that there is large gluon content in $X(1835)$. Also some authors suggest that the two structures observed in $J/\psi \to \gamma p\overline{p}$ and $J/\psi \to \gamma\eta'\pi^+\pi^-$ are not the same state, and identify $X(1835)$ as the η's second radial excitation [15]. Obviously, more theoretical and experimental efforts are needed to determine whether $X(1835)$ exists or not, and to be sure that $X(1835)$ is a $p\overline{p}$ bound state or glueball or something else. Motivated by solving the puzzles mentioned above and getting the information about the structure of $X(1835)$, we investigate the productions of $X(1835)$ in Υ and J/ψ decays in this work. The production of $X(1835)$ may provide significant information on the structure of $X(1835)$.

So far, the experiments strongly indicate that $X(1835)$ is almost uniquely produced in J/ψ radiative decays and it has large coupling with $p\overline{p}$ and $\eta'\pi\pi$. Whatever $X(1835)$ is a glueball or $p\overline{p}$ bound state or something else, it must meet these two significant experimental facts. In this work the possibility of $X(1835)$ as a baryonium with sizable gluon content is investigated. In this perspective, the puzzles mentioned above can be answered naturally.

The paper is organized as follows: In sect. 2, we suggest $X(1835)$ as a baryonium with sizable gluon content, whose gluon content is similar to that of η'. In this perspective, we can easily understand the reasons why $\Upsilon(1S) \to \gamma X(1835)$ and $J/\psi \to \omega X(1835)$ are not seen at the present stage. In sect. 3 we conjecture the existence of pseudoscalar baryonium nonet and study its production in J/ψ decay in a model-independent way. Finally, we briefly summarize the results and give some discussions.

2 The possible structure of X(1835) and Υ(1S) → γX(1835), J/ψ → ωX(1835)

The production of $X(1835)$ in J/ψ radiative decay $J/\psi \to \gamma\eta'\pi^+\pi^-$ may indicate that there is large gluon content in $X(1835)$, as is shown in refs. [10,13]. Also $J/\psi \to \gamma + gg, gg \to$ *hadrons* provide an important search ground for the glueball [16], some people suggest that $X(1835)$ is a 0^{-+} glueball. However, the lowest pseudoscalar glueball mass is 2.1–2.5 GeV from the quenched lattice approach [17], and 2.05 ± 0.19 GeV, 2.2 ± 0.2 GeV in QCD sum rules [18] and it seems difficult to explain the large mass difference between 1835 MeV and the theoretical prediction mass. On the other hand, even if $X(1835)$ is a pure glueball, it would mix with other mesonic states, such as $\eta(1440)$, $\eta(1295)$ and $\eta_c(1S)$.

Furthermore, in ref. [11] we show that $X(1835)$ can be possibly a baryonium and the relative large mass defect can be produced. In ref. [10], we pointed out that there is sizeable gluon content in the skyrmion-baryonium $X(1860)$ (*i.e.*, $X(1835)$) by discussing the baryonium decay through baryon-antibaryon annihilation in the Skyrme model. Distinguishing from the naive (or old-fashioned) baryonium in the Fermi-Yang–type models [7,19–21], the skyrmion-baryonium is constructed in the model inspired by QCD, and therefore the gluon inside the baryonium will play an important role in the baryonium physics, *e.g.*, the baryonium decays and productions. Therefore, the skyrmion-baryonium belongs to a sort of baryonium with sizable gluon content. We address that in the naive baryonium model framework it is difficult to simultaneously explain the large branching fraction $X(1835) \to p\overline{p}$, $X(1835) \to \eta'\pi^+\pi^-$. The gluon content in $X(1835)$ should play essential role in the $X(1835)$ decay [10]. So it is natural to treat $X(1835)$ as a baryonium with sizable gluon content, which looks like η' in some sense, and mainly belongs to a $SU(3)$ flavor singlet.

In the following two subsections, we will start with this view to examine the branching fractions of $\Upsilon(1S) \to \gamma X(1835)$ and $J/\psi \to \omega X(1835)$, respectively. We will show that the branching fractions of both $\Upsilon(1S) \to \gamma X(1835)$ and $J/\psi \to \omega X(1835)$ are much smaller comparing to that of $J/\psi \to \gamma X(1835)$. We will also predict that the branching fraction of $J/\psi \to \rho X(1835)$ is very small, so we conclude that the process with visible $X(1835)$ may only be the J/ψ radiative decay at the present stage.

2.1 Υ(1S) → γX(1835)

According to Novikov *et al.* [22], for the J/ψ radiative decay, the photon is emitted by the c quark with a subsequent annihilation of the $c\overline{c}$ into light quarks through the effect of the $U(1)_A$ anomaly. The creation of the corresponding light quarks is controlled by the gluonic matrix element $\langle \frac{\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}|P_i\rangle$ (P_i is a pseudoscalar, it can be η, η', and $X(1835)$ and so on). Photon emission from the light quarks is negligible as can be seen from the smallness of the $J/\psi \to \gamma\pi$ decay width, this mechanism leads to the following width for the J/ψ radiative decay into the pseudoscalar P_i

$$\begin{aligned}\Gamma(J/\psi \to \gamma P_i) = \frac{2^5}{5^2 3^8}\pi e_c^2 \alpha_{em}^3 K[J/\psi\gamma P_i]^3 \\ \times \left(\frac{M_{J/\psi}}{m_c^2}\right)^4 \frac{\left|\langle \frac{\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}|P_i\rangle\right|^2}{\Gamma(J/\psi \to e^+e^-)}\,,\end{aligned} \tag{1}$$

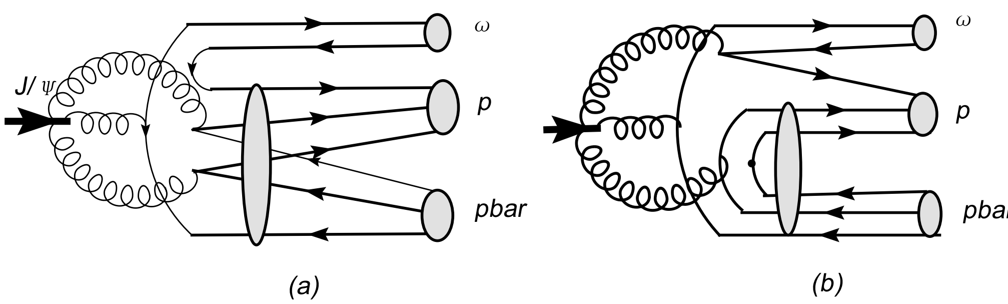

Fig. 1. A schematic diagram for $J/\psi \to \omega X(1835) \to p\overline{p}$ in a mechanism with a $^3P_0$ quark pair created in two configurations.

where $K[J/\psi\gamma P_i]$ is the momentum of the pseudoscalar P_i in the J/ψ rest frame, and $K[J/\psi\gamma P_i] = \frac{M_{J/\psi}}{2}(1-\frac{M^2_{P_i}}{M^2_{J/\psi}})$. Then, the ratio of the branching fractions between $J/\psi \to \gamma X(1835)$ and $J/\psi \to \gamma\eta'$ is

$$\frac{Br(J/\psi \to \gamma X(1835))}{Br(J/\psi \to \gamma\eta')} = \frac{K[J/\psi\gamma X(1835)]^3}{K[J/\psi\gamma\eta']^3}\frac{\left|\left\langle \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}|X(1835)\right\rangle\right|^2}{\left|\left\langle \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}|\eta'\right\rangle\right|^2}. \quad (2)$$

It is straightforward to extend the anomaly dominance to the case of the $\Upsilon(1S)$ radiative decay [23,24]. Then, we have

$$\frac{Br(\Upsilon(1S) \to \gamma X(1835))}{Br(\Upsilon(1S) \to \gamma\eta')} = \frac{K[\Upsilon(1S)\gamma X(1835)]^3}{K[\Upsilon(1S)\gamma\eta']^3}\frac{\left|\left\langle \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}|X(1835)\right\rangle\right|^2}{\left|\left\langle \frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}|\eta'\right\rangle\right|^2}. \quad (3)$$

From eq. (2) and eq. (3), we get

$$Br(\Upsilon(1S) \to \gamma X(1835)) = \frac{K[\Upsilon(1S)\gamma X(1835)]^3}{K[\Upsilon(1S)\gamma\eta']^3}\frac{K[J/\psi\gamma\eta']^3}{K[J/\psi\gamma X(1835)]^3} \times \frac{Br(\Upsilon(1S) \to \gamma\eta')}{Br(J/\psi \to \gamma\eta')}Br(J/\psi \to \gamma X(1835)). \quad (4)$$

For the $\Upsilon(1S)$ radiative decay $\Upsilon(1S) \to \gamma\eta'$, only the upper limit has been obtained, which is $Br(\Upsilon(1S) \to \gamma\eta') < 1.6 \times 10^{-5}$ at 90% confidental level [25]. Using again the Particle Data Group's value $Br(J/\psi \to \gamma\eta') = (4.31 \pm 0.30) \times 10^{-3}$ [25], and substituting it into eq. (4), we obtain

$$Br(\Upsilon(1S) \to \gamma X(1835)) < 9.22 \times 10^{-3} Br(J/\psi \to \gamma X(1835)). \quad (5)$$

Thus, since BES has already determined $Br(J/\psi \to \gamma X(1835))Br(X(1835) \to p\overline{p}) = (7.0 \pm 0.4^{+1.9}_{-0.8}) \times 10^{-5}$ [2], and by eq. (5), we finally get a reasonable estimation:

$$Br(\Upsilon(1S) \to \gamma X(1835))Br(X(1835) \to p\overline{p}) < 6.45 \times 10^{-7}. \quad (6)$$

This estimation is compatible with CLEO Collaboration's result of $Br(\Upsilon(1S) \to \gamma X(1835))Br(X(1835) \to p\overline{p}) < 5 \times 10^{-7}$. So it is not surprising that the CLEO Collaboration do not see the signal of $X(1835)$ in the $\Upsilon(1S)$ radiative decay, and it does not mean that $X(1835)$ seen by BES is an experimental artifact.

2.2 $J/\psi \to \omega X(1835)$

In this subsection, we examine the branching fraction of $J/\psi \to \omega X(1835)$. Since $Br(J/\psi \to \gamma X(1835))$ is rather large [1,2], one could expect $Br(J/\psi \to \omega X(1835))$ may also be reasonably large too, or at least be visible at present stage. In this way the existence of $X(1835)$ may be rechecked in the non-radiative decay channel of J/ψ. However, this is only a naive conjecture, and experimentally there is not yet any data on this branching fraction. So, a theoretical estimation on $Br(J/\psi \to \omega X(1835))$ is necessary. Our estimations in this subsection are still based on the baryonium picture discussed in the above.

Unlike the radiative decays $J/\psi \to \gamma X(1835)$ and $\Upsilon(1S) \to \gamma X(1835)$, where the gluon component plays an important role due to the $U_A(1)$ anomaly in the $J/\psi \to \omega X(1835)$ decay, the processes to which the $U_A(1)$ anomaly contributes are suppressed, and the baryonic component dominates this process, the same is true for $J/\psi \to \omega\eta'$. We think that the decay process $J/\psi \to \omega X(1835)$ proceeds via two steps as illustrated in fig. 1. In the first step, the $c\overline{c}$ pair annihilates into three gluons, followed by the materialization of each gluon into a pair of quark-antiquark, this process can be calculated from perturbative QCD to the lowest order. Also a pair of quark-antiquark is created from the vacuum, and this process can be described by the quark pair creation model (the $^3P_0$ model). Then, in the second step the quarks and the antiquarks combine to form ω and $X(1835)$. Here, the non-perturbative dynamics is included by the hadron's wave functions in the naive quark model.

The quark pair creation model, which describes the process that a pair of quark-antiquark with quantum number $J^{PC} = 0^{++}$ is created from vacuum, was first proposed by Micu [26] in 1969. In the 1970s, this model was developed by Yaouanc *et al.* [27–30] and applied to study hadron decays extensively. The $^3P_0$ quark pair creation model has proven to be a successful mechanism for describing the strong decay of light mesons [31–33]. It has

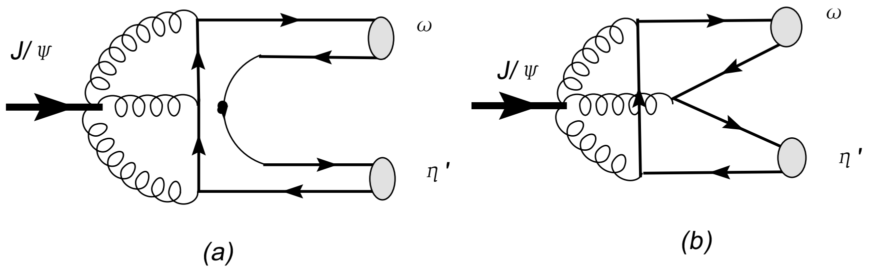

Fig. 2. A schematic diagram for $J/\psi \to \omega\eta'$ decay mechanism: (a) a $^3P_0$ quark pair created; (b) no $^3P_0$ quark pair created.

also been shown that the $^3P_0$ quark pair creation mechanism may play an important role for some exclusive decay in the charmonium sector [34–36]. In the $^3P_0$ model, the created quark pairs with any color and any flavor can be generated anywhere in space, but only those whose color-flavor wave functions and spatial wave functions overlap with those of outgoing hadrons can make a contribution to the final decay width. The Hamiltonian for creating a quark pair can be defined in the $^3P_0$ model in terms of quark and antiquark creation operators $b^\dagger$ and $d^\dagger$ [33],

$$H_I = \sum_{i,j,\alpha,\beta,s,s'} \int \mathrm{d}^3k\, g_I[\overline{u}(\mathbf{k},s)v(-\mathbf{k},s')] \times b^\dagger_{\alpha,i}(\mathbf{k},s)d^\dagger_{\beta,j}(-\mathbf{k},s')\delta_{\alpha\beta}\hat{C}_I, \quad (7)$$

where $\alpha(\beta)$ and $i(j)$ are the flavor and color indices of the created quarks (antiquarks), and $u(\mathbf{k},s)$ and $v(\mathbf{k}',s')$ are free Dirac spinors for quarks and antiquarks, respectively. $\hat{C}_I = \delta_{ij}$ is the color operator for $q\overline{q}$ and g_I is the strength of the decay interaction, which is assumed as a constant in these processes. In the non-relativistic limit, g_I can be related to γ, the strength of the conventional $^3P_0$ model, by $g_I = 2m_q\gamma$ [33]. In order to cancel the large uncertainty in g_I and the overall constant dependence, we will calculate the ratio $\frac{\Gamma(J/\psi\to\omega X(1835))}{\Gamma(J/\psi\to\omega\eta')}$ in the following. The process $J/\psi \to \omega\eta'$ is schematically shown in fig. 2, where the electromagnetic decay process is not shown. For the J/ψ decaying into hadrons, the ratio between the hadronic decay width and the electromagnetic decay width is about 5 [37], *i.e.*, $\frac{\Gamma(J/\psi\to ggg\to hadrons)}{\Gamma(J/\psi\to\gamma\to hadrons)} \simeq 5$. Thus we can include the contribution of the electromagnetic decay to $J/\psi \to \omega\eta'$ though the above ratio.

For the $J/\psi \to PV$ decays, the parity transformation is conserved, and the transitional amplitude squared is $\sum_\Lambda |M(\Lambda)|^2 = (1+\cos^2\theta)|A_1|^2$, where Λ is the J/ψ helicity, which is taken as $\Lambda = \pm 1$ if it is produced from e^+e^- annihilations, A_1 is the helicity amplitude with vector meson helicity equal to 1, and θ is the polar angle of the outgoing meson. The decay width $\Gamma = \frac{|\mathbf{P}|}{6\pi M^2_{J/\psi}}|A_1|^2$, here $\mathbf{P}$ is the momentum of out-going mesons.

2.2.1 $J/\psi \to \omega X(1835) \to \omega p\bar{p}$

The color factors for fig. 1 are:

– color factor for fig. 1(a), $c_a = \frac{5}{54}$;
– color factor for fig. 1(b), $c_b = \frac{5}{432}$ (to be negligible).

The amplitude can be obtained according to the standard Feynman rules with the quark pair creation Hamiltonian included, which is expressed as follows:

$$\begin{aligned} &T_{\Lambda,s}(J/\psi \to \omega X(1835)) \\ &= C_0 c_a \alpha_s^{3/2}\langle\phi_\omega\phi_X|\overline{u}(\mathbf{p}_1,s_1)\gamma_\mu v(\mathbf{q}_1,\bar{s}_1) \\ &\times\overline{u}(\mathbf{p}_2,s_2)\gamma_\nu v(\mathbf{q}_2,\bar{s}_2)\overline{u}(\mathbf{p}_3,s_3)\gamma_\rho v(\mathbf{q}_3,\bar{s}_3)\epsilon_\psi^{(\Lambda)\lambda} \\ &\times\frac{g_{\mu\lambda}g_{\nu\rho}+g_{\nu\lambda}g_{\mu\rho}+g_{\rho\lambda}g_{\mu\nu}}{k_1^2k_2^2k_3^2} g_I\overline{u}(\mathbf{p}_4,s_4)v(\mathbf{q}_4,\bar{s}_4)|\phi_{J/\psi}\rangle, \end{aligned} \quad (8)$$

where C_0 is the coupling constant for $J/\psi \to ggg$, and α_s is the strong-coupling constant. k_i is the gluonic momentum, and the normalization of Dirac spinor is taken as $\overline{u}u = -\overline{v}v = m/E$. ϕ_ω, ϕ_X and $\phi_{J/\psi}$ represent the wave functions of ω, $X(1835)$ and J/ψ, respectively. And the helicity amplitude is:

$$\begin{aligned} &A_{\Lambda,s}(J/\psi \to \omega X(1835)) = \\ &\int \prod_{i=1,4} \frac{\mathrm{d}^3\mathbf{q}_i}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_i}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{t}_1}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{t}_2}{(2\pi)^3}\, T_{\Lambda,s}(J/\psi \to \omega X(1835)) \\ &\times(2\pi)^3\delta^3(\mathbf{p}_\omega - \mathbf{p}_1 - \mathbf{q}_4)(2\pi)^3\delta^3(\mathbf{p}_2+\mathbf{p}_3+\mathbf{p}_4-\mathbf{t}_1) \\ &\times(2\pi)^3\delta^3(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3-\mathbf{t}_2)(2\pi)^3\delta^3(\mathbf{t}_1+\mathbf{t}_2-\mathbf{p}_X) \\ &\times(2\pi)^3\delta^3(\mathbf{p}_4+\mathbf{q}_4). \end{aligned} \quad (9)$$

Here, Λ and s denote the helicity of J/ψ and ω, respectively.

2.2.2 $J/\psi \to \omega\eta'$

The color factors for fig. 2 are:

– color factor for fig. 2(a): $c'_a = \frac{5\sqrt{3}}{54}$;
– color factor for fig. 2(b): $c'_b = \frac{5\sqrt{3}}{216}$ (negligible).

The corresponding helicity amplitude is

$$\begin{aligned} &A_{\Lambda,s}\big(J/\psi \to \omega\eta'\big) = \\ &\int \frac{\mathrm{d}^4k_1}{(2\pi)^4}\frac{\mathrm{d}^4k_2}{(2\pi)^4}\frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{q}_1}{(2\pi)^3} T_{\Lambda,s}\big(J/\psi \to \omega\eta'\big) \\ &\times(2\pi)^3\delta^3\big(\mathbf{q}_\omega - \mathbf{p}_1 + \mathbf{p}_2\big)(2\pi)^3\delta^3\big(\mathbf{q}_{\eta'} - \mathbf{q}_1 - \mathbf{p}_2\big), \end{aligned} \quad (10)$$

Table 1. Numerical results of $\frac{\Gamma(J/\psi\to\omega X(1835))}{\Gamma(J/\psi\to\omega\eta')}$ corresponding to the different set of harmonic-oscillator parameters β and β_X, with the contribution of the electromagnetic process included, where the quark masses are taken as $m_u = m_d = 0.31\,\mathrm{GeV}$.

β(GeV)	β_X(GeV)				Average of $\frac{\Gamma(J/\psi\to\omega X)}{\Gamma(J/\psi\to\omega\eta')}$
	0.15	0.20	0.25	0.30	
0.36	6.2	5.9	5.0	4.1	5.2 ± 1.0
0.40	0.15	0.13	0.12	0.10	0.12 ± 0.02
0.46	0.006	0.006	0.005	0.005	0.004 ± 0.0008
0.52	5.9×10^{-4}	5.6×10^{-4}	4.8×10^{-4}	4.0×10^{-4}	$(5.0\pm0.9)\times10^{-4}$

where $T_{\Lambda,s}(J/\psi\to\omega\eta')$ is

$$\begin{aligned}T_{\Lambda,s}(J/\psi\to\omega\eta') =& C_0 c'_a \alpha_s^{3/2}\langle\phi_\omega(q_\omega,s)\phi_{\eta'}(q_{\eta'})|\overline{u}(p_1,s_1)\\ &\times\gamma_\mu\frac{1}{\not p_1-\not k_1-m}\gamma_\nu\frac{1}{\not q_1-\not k_2-m}\gamma_\rho v(q_1,\bar{s}_1)\\ &\times\epsilon_\psi^{(\Lambda)\lambda}\frac{g_{\mu\lambda}g_{\nu\rho}+g_{\nu\lambda}g_{\mu\rho}+g_{\rho\lambda}g_{\mu\nu}}{k_1^2k_2^2k_3^2}\\ &\times g_I\overline{u}(p_2,s_2)v(-p_2,\bar{s}_2)|\phi_{J/\psi}\rangle. \qquad (11)\end{aligned}$$

For simplicity, we make use of the on-shell approximation, *i.e.*, $\frac{1}{k_1^2k_2^2}\to-2\pi^2\delta(k_1^2)\delta(k_2^2)$, then the following relation holds:

$$\begin{aligned}\int\frac{\mathrm{d}^4k_1\mathrm{d}^4k_2}{k_1^2k_2^2k_3^2}&\to-\frac{\pi^2}{2}\int_0^{M_\psi}\mathrm{d}k_1^0\int_0^{M_\psi-k_1^0}\mathrm{d}k_2^0\\ \times\int\mathrm{d}\Omega_1\mathrm{d}\Omega_2&\frac{k_1^0k_2^0}{M_\psi^2-2k_1^0M_\psi-2k_2^0M_\psi+2k_1\cdot k_2}. \qquad (12)\end{aligned}$$

2.2.3 Numerical results

The spin-flavor wave functions of the mesons ω and η' are well known in the quark model, and the spatial wave function is taken as the simple harmonic-oscillator wave function, *i.e.*, $\phi(\mathbf{k})=\frac{(2\pi)^{3/2}}{(\pi\beta^2)^{3/4}}e^{-\mathbf{k}^2/2\beta^2}$. Since $X(1835)$ is assumed as a baryonium with $J^{PC}=0^{-+}$, its spatial wave function is the product of the proton spatial wave function, the antiproton spatial wave function and the relative spatial wave function between them. It is expressed as follows:

$$\phi_X=\phi_p(\mathbf{p}_\rho,\mathbf{p}_\lambda)\phi_{\overline{p}}(\mathbf{q}_\rho,\mathbf{q}_\lambda)\phi_{p\overline{p}}(\mathbf{t}_1-\mathbf{t}_2), \qquad (13)$$

where $\phi_p(\mathbf{p}_\rho,\mathbf{p}_\lambda)=\frac{(2\pi)^{3/2}}{(\pi\beta^2)^{3/2}}e^{-(\mathbf{p}_\rho^2+\mathbf{p}_\lambda^2)/2\beta^2}$, and $\phi_{p\overline{p}}(\mathbf{t}_1-\mathbf{t}_2)$ is of the same formalism as the simple harmonic-oscillator wave function with $\mathbf{p}_\rho=\frac{1}{\sqrt{6}}(\mathbf{p}_2+\mathbf{p}_3-2\mathbf{p}_4)$, $\mathbf{q}_\rho=\frac{1}{\sqrt{6}}(\mathbf{q}_1+\mathbf{q}_2-2\mathbf{q}_3)$, $\mathbf{p}_\lambda=\frac{1}{\sqrt{2}}(\mathbf{p}_2-\mathbf{p}_3)$, $\mathbf{q}_\lambda=\frac{1}{\sqrt{2}}(\mathbf{q}_1-\mathbf{q}_2)$, $\mathbf{t}_1=\mathbf{p}_2+\mathbf{p}_3+\mathbf{p}_4$ and $\mathbf{t}_2=\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3$. The spin-flavor wave function is the following:

$$\begin{aligned}&\frac{1}{2\sqrt{2}}\Big\{\Big[\chi_p^\rho(\uparrow)\phi_p^\rho+\chi_p^\lambda(\uparrow)\phi_p^\lambda\Big]\Big[\chi_{\overline{p}}^\rho(\downarrow)\phi_{\overline{p}}^\rho+\chi_{\overline{p}}^\lambda(\downarrow)\phi_{\overline{p}}^\lambda\Big]\\ &-\Big[\chi_p^\rho(\downarrow)\phi_p^\rho+\chi_p^\lambda(\downarrow)\phi_p^\lambda\Big]\Big[\chi_{\overline{p}}^\rho(\uparrow)\phi_{\overline{p}}^\rho+\chi_{\overline{p}}^\lambda(\uparrow)\phi_{\overline{p}}^\lambda\Big]\Big\}, \qquad (14)\end{aligned}$$

where ϕ_p^ρ and ϕ_p^λ are the ρ-type and λ-type nucleon flavor wave functions, respectively, and similarly for $\chi_{p/\overline{p}}^\rho$ and $\chi_{p/\overline{p}}^\lambda$.

There are four parameters to be determined in our calculation, *i.e.*, the quark masses m_u, m_d, the harmonic-oscillator parameter β for hadrons and $X(1835)$. The quark masses are taken as $m_u=m_d=0.31\,\mathrm{GeV}$. In most calculations in quark model, the harmonic-oscillator parameter is fitted to the hadron decay width, which gives $\beta=0.4\,\mathrm{GeV}$ [32,33]. The harmonic-oscillator parameter of $X(1835)$ is determined by assuming that the radius of $p\overline{p}$ is about 1–2 fm, which corresponds to the parameter $\beta_X=0.15$–$0.30\,\mathrm{GeV}$. The ratio of $\Gamma(J/\psi\to\omega X(1835))/\Gamma(J/\psi\to\omega\eta')$ is calculated in terms of different sets of harmonic-oscillator parameters β and β_X as listed in table 1. It is clear to see that the ratio is very sensitive to the parameter β and not sensitive to β_X.

As most quark model studies on the meson decays, we use the parameters $m_u=m_d=0.31$, $\beta=0.4\,\mathrm{GeV}$ as our favorable parameters. In our calculation, the uncertainties are from the parameter β_X, the ignored decay modes depressed by color factor and the accuracy of the numerical calculation. From our estimation, the uncertainty from β_X within our setting range is about 20%. The contribution from fig. 1(b) and fig. 2(b) is of the same order as that from fig. 1(a) and fig. 2(a), respectively. The color-depressed decay modes will bring in an uncertainty of about 6%, the uncertainty of the numerical evaluation is about 8%, then the total uncertainty is about 22%. Including these uncertainties, we predict the ratio $\Gamma(J/\psi\to\omega\eta')/\Gamma(J/\psi\to\omega X)=0.12\pm0.02$. Using the PDG value $Br(J/\psi\to\omega\eta')=(1.67\pm0.25)\times10^{-4}$, we predict that $Br(J/\psi\to\omega X(1835))=(2.00\pm0.35)\times10^{-5}$.

Comparing this result with $Br(J/\psi\to\gamma X(1835))\sim(0.5$–$2)\times10^{-3}$ [1], we see that the production rate of $X(1835)$ in the process $J/\psi\to\omega X(1835)$ is less than that in $J/\psi\to\gamma X(1835)$ of about two orders. Therefore, it is not surprising that the signal of $J/\psi\to\omega X(1835)$ has not been seen by BES or other laboratories so far. In other words, the absence of the signal in the decay $J/\psi\to\omega X(1835)$ at present cannot be thought as an evidence against the existence of $X(1835)$.

Using BES's estimations of $Br(X(1835)\to p\overline{p})\sim(4$–$14)\%$ and $Br(X(1835)\to\eta'\pi^+\pi^-)\sim3Br(X(1835)\to p\overline{p})$ [1,3], we get further two useful estimations about the

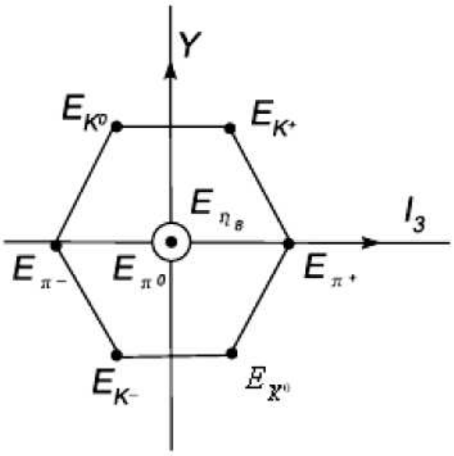

Fig. 3. The weight diagram for the pseudoscalar baryonium octet.

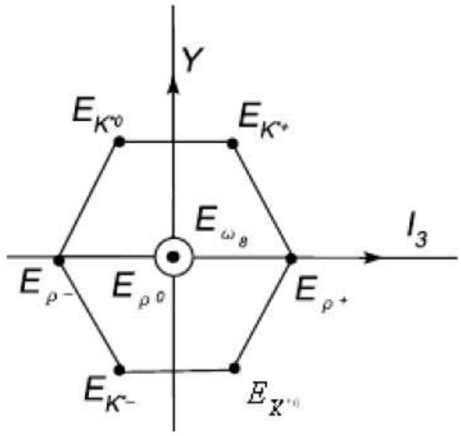

Fig. 4. The weight diagram for the vector baryonium octet.

product branching fractions:

$$8.00 \times 10^{-7} < Br(J/\psi \to \omega X(1835))Br\big(X(1835) \to p\overline{p}\big) < 2.80 \times 10^{-6}, \qquad (15)$$

$$2.40 \times 10^{-6} < Br\big(J/\psi \to \omega X(1835)\big) \times Br\big(X(1835) \to \eta'\pi^+\pi^-\big) < 8.40 \times 10^{-6}. \quad (16)$$

Comparing them with the data [1] $Br(J/\psi \to \gamma X)Br(X \to p\overline{p}) = (7.0 \pm 0.4^{+1.9}_{-0.8}) \times 10^{-5}$ and $Br(J/\psi \to \gamma X(1835))Br(X(1835) \to \pi^+\pi^-\eta') = (2.2 \pm 0.4(\text{stat.}) \pm 0.4(\text{syst.})) \times 10^{-4}$ respectively, we see that also both $Br(J/\psi \to \omega X(1835))Br(X(1835) \to p\overline{p})$ and $Br(J/\psi \to \omega X(1835))Br(X(1835) \to \eta'\pi^+\pi^-)$ are very small. So, the signal of $X(1835)$ is very difficult, if not impossible, to be observed in the process $J/\psi \to \omega X(1835)$ with $X(1835) \to p\overline{p}$ or $X(1835) \to \eta'\pi^+\pi^-$.

Finally, we discuss the production of $X(1835)$ in the process $J/\psi \to \rho X(1835)$. In this process the G-parity is not conserved, and it proceeds through a virtual photon $c\overline{c} \to \gamma^*$. Contributions from the isospin-violating part of QCD are supposedly very small. Furthermore, the masses of ρ and ω are approximately equal, so $Br(J/\psi \to \rho X(1835)) < Br(J/\psi \to \omega X(1835))$ (the same holds true for $J/\psi \to \omega\eta'$ and $J/\psi \to \rho\eta'$, that is $Br(J/\psi \to \omega\eta') > Br(J/\psi \to \rho\eta')$). This means that we also cannot see $X(1835)$ in the process $J/\psi \to \rho X(1835)$.

After the above analysis, we conclude that comparing to $Br(J/\psi \to \gamma X(1835))$, the branching fractions of $J/\psi \to VX(1835)$, with V being ω or ρ, are heavily suppressed due to $X(1835)$'s special structure. The search for the $X(1835)$ in these decays seems impossible at the present stage.

3 Baryonium nonet and its production in a model-independent way

The BES Collaboration has observed not only the $p\overline{p}$ enhancement [1,2], but also the $p\overline{\Lambda}$ enhancement [38]. These two states can belong to flavor **1**-*plet*, **8**-*plet*, **10**-*plet*, $\overline{\mathbf{10}}$-*plet*, or **27**-*plet*. It seems that at least a baryonium nonet exists. Theoretically, we have predicted the existence of such a baryonium nonet in ref. [11]. The baryonium nonet was also suggested from the Fermi-Yang-Sakata model in ref. [8]. The nonet can be a pseudoscalar or a vector multiplet [8,11], and the corresponding weight diagrams are shown in fig. 3 and fig. 4, respectively. The pseudoscalar and the vector enhancement octet are, respectively, denoted by E_{P_i} and E_{V_i} $(i = 1 \cdots 8)$ as follows:

$$\begin{aligned}
&E_{\pi^\pm} = \frac{1}{\sqrt{2}}\big(E_{P_1} \mp iE_{P_2}\big),\\
&E_{\pi^0} = E_{P_3}, \qquad E_{K^\pm} = \frac{1}{\sqrt{2}}\big(E_{P_4} \mp iE_{P_5}\big),\\
&E_{K^0} = \frac{1}{\sqrt{2}}\big(E_{P_6} - iE_{P_7}\big),\\
&E_{\overline{K}^0} = \frac{1}{\sqrt{2}}\big(E_{P_6} + iE_{P_7}\big), \qquad E_{\eta_8} = E_{P_8}.
\end{aligned} \qquad (17)$$

It is useful to add the singlet to the octet E_P by defining $E_{\eta_1} = E_{P_0}$, thereby creating the nonet $E_P = (E_{P_0}, E_{P_i})$. If the pseudoscalar glueball and radially exciting states are ignored, the physics states $E_{\eta'}$ and E_η are a mix of E_{η_8} and E_{η_0} with the mixing angle φ_P:

$$\begin{aligned}
E_{\eta_8} &= \cos\varphi_P E_\eta + \sin\varphi_P E_{\eta'},\\
E_{\eta_1} &= -\sin\varphi_P E_\eta + \cos\varphi_P E_{\eta'}.
\end{aligned} \qquad (18)$$

Similarly, for the vector enhancement nonet $E_V = (E_{\omega_1}, E_{V_i})$, the physics states E_ω and E_ϕ are a mix of E_{ω_8} and E_{ω_1} with the mixing angle φ_V:

$$\begin{aligned}
E_{\omega_8} &= \cos\varphi_V E_\phi + \sin\varphi_V E_\omega,\\
E_{\omega_1} &= -\sin\varphi_V E_\phi + \cos\varphi_P E_\omega.
\end{aligned} \qquad (19)$$

We identify the $p\overline{p}$ enhancement $X(1835)$ as the state $E_{\eta'}$, while the $p\overline{\Lambda}$ enhancement should be E_{K^+} or $E_{K^{*+}}$, and $E_{K^{*+}}$ is favored over E_{K^+} from the analysis of ref. [11].

We can consider the flavor $SU(3)$ breaking by choosing a nonet pointing to a fixed direction in $SU(3)$ space, particularly for the desired breaking. We will consider two types of $SU(3)$ breaking, first, $SU(3)$ is broken due to $m_s \neq m_u, m_d$ ($m_u = m_d$ is assumed), the quark mass term is $m_d(d\overline{d}+u\overline{u})+m_s s\overline{s} = m_0 q\overline{q}+\sqrt{\frac{1}{3}}(m_d-m_s)\overline{q}\lambda_8 q$, where $q=(u,d,s)$ and $m_0=\frac{1}{3}(2m_d+m_s)$. We can see that this $SU(3)$ breaking corresponds to a nonet $\mathbf{M}$, pointing to the 8th direction, *i.e.*, $M^a=\delta^{a8}$. Second, the electromagnetic effects violate the $SU(3)$ invariance, since the photon coupling to quarks is $\frac{2}{3}\overline{u}\gamma_\mu u-\frac{1}{3}\overline{d}\gamma_\mu d-\frac{1}{3}\overline{s}\gamma_\mu s=\frac{1}{2}\overline{q}\gamma_\mu(\lambda_3+\frac{\lambda_8}{\sqrt{3}})q$. This symmetry-breaking effect corresponds to a nonet $\mathbf{E}$, given by $E^a=\delta^{a3}+\sqrt{\frac{1}{3}}\delta^{a8}$. In the following, we will consider the process of $J/\psi\to E_P P$, that means that J/ψ decays into a pseudoscalar baryonium $(E_\pi, E_K, E_\eta, E_{\eta'})$ and a pseudoscalar (π, K, η, η'), the process $J/\psi\to E_P V$, *i.e.*, J/ψ decays to a pseudoscalar baryonium $(E_\pi, E_K, E_\eta, E_{\eta'})$ and a vector $(\rho, K^*, \omega, \phi)$ and the process $J/\psi\to E_V P$, *i.e.*, J/ψ decays to a vector baryonium $(E_\rho, E_{K^*}, E_\omega, E_\phi)$ and a pseudoscalar (π, K, η, η') in a model-independent way via $SU(3)$ symmetry with the effects of electromagnetic and mass breaking of the $SU(3)$ symmetry included [39,40]. Since the phase space factor is proportional to the cube of the final three-momentum, we define the reduced branching fraction $\widetilde{Br}(J/\psi\to E_P P)=Br(J/\psi\to E_P P)/P_P^3$, here P_P is the momentum of the pseudoscalar P in the J/ψ rest frame, and the reduced branching fractions $\widetilde{Br}(J/\psi\to E_P V)$, $\widetilde{Br}(J/\psi\to E_V P)$ are defined in the same way.

3.1 $J/\psi\to E_P P$

These processes occur completely due to the $SU(3)$ breaking effects; we can construct the charge conjugation invariant and the $SU(3)$ invariant effective Lagrangian involving the symmetry-breaking nonet $\mathbf{E}$ or $\mathbf{M}$, which may be written as follows:

$$\mathcal{L}_{eff}=f^{abc}\Psi^\mu E_{P_a}\overset{\leftrightarrow}{\partial_\mu}P_b\big(g_M M^c+g_E E^c\big). \qquad (20)$$

Here the new parameters g_M and g_E parametrize the $SU(3)$ breaking effects. From eq. (19) and eq. (20) we can obtain the following reduced branching fractions:

$$\widetilde{Br}\big(J/\psi\to E_{\pi^+}\pi^-\big)=\widetilde{Br}\big(J/\psi\to E_{\pi^-}\pi^+\big)=|g_E|^2,$$
$$\widetilde{Br}\big(J/\psi\to E_{K^+}K^-\big)=\widetilde{Br}\big(J/\psi\to E_{K^-}K^+\big)=\left|\frac{\sqrt{3}}{2}g_M+g_E\right|^2,$$
$$\widetilde{Br}\Big(J/\psi\to E_{K^0}\overline{K}^0\Big)=\widetilde{Br}\big(J/\psi\to E_{\overline{K}^0}K^0\big)=\frac{3}{4}|g_M|^2,$$
$$\widetilde{Br}\big(J/\psi\to E_{\pi^0}\pi^0\big)=\widetilde{Br}\big(J/\psi\to E_\eta\pi^0\big)=\widetilde{Br}\big(J/\psi\to E_{\eta'}\pi^0\big)=0. \qquad (21)$$

The last formula means that the process $J/\psi\to\pi^0X(1835)$, $X(1835)\to p\overline{p}$ should be forbidden, which is indeed not observed [2], and it is forbidden because of C-parity.

3.2 $J/\psi\to E_P V$

Following the same way as in the discussion of $J/\psi\to E_P P$, we construct the charge conjugation invariant, the $SU(3)$ invariant effective Lagrangian including the symmetry-breaking nonet $\mathbf{E}$ and $\mathbf{M}$, which may be written as follows:

$$\mathcal{L}_{eff}=\varepsilon_{\mu\nu\alpha\beta}F_\Psi^{\mu\nu}\Bigg\{g_8F_{V_a}^{\alpha\beta}E_{P_a}+g_1F_{\omega_1}^{\alpha\beta}E_{\eta_1}+\left[g_{M,88}d^{abc}F_{V_a}^{\alpha\beta}E_{P_b}M^c+\sqrt{\frac{2}{3}}g_{M,18}F_{V_a}^{\alpha\beta}M^aE_{\eta_1}+\sqrt{\frac{2}{3}}g_{M,81}F_{\omega_1}^{\alpha\beta}M^aE_{P_a}\right]+\left[g_{E,88}d^{abc}F_{V_a}^{\alpha\beta}E_{P_b}E^c+\sqrt{\frac{2}{3}}g_{E,18}F_{V_a}^{\alpha\beta}E^aE_{\eta_1}+\sqrt{\frac{2}{3}}g_{E,81}F_{\omega_1}^{\alpha\beta}E^aE_{P_a}\right]\Bigg\}, \qquad (22)$$

where $F_\Psi^{\mu\nu}$ is the strength of the J/ψ field with $F_\Psi^{\mu\nu}=\partial^\mu\Psi^\nu-\partial^\nu\Psi^\mu$, and $F_{\omega_1}^{\alpha\beta}$, $F_{V_a}^{\alpha\beta}$ are, respectively, the field strength of the vector field ω_1 and V_a. We assume the nonet symmetry holds true within a reasonable approximation, which relates the octet to the singlet, then we have the relations $g_8=g_1\equiv g$, $g_{M,88}=g_{M,81}=g_{M,18}\equiv g'_M$ and $g_{E,88}=g_{E,81}=g_{E,18}\equiv g'_E$. We take the parameters g, g_M and g_E to be small and calculate the $SU(3)$ breaking to first order in these parameters. From eq. (17), eq. (18), the ω-ϕ "ideal" mixing and the Lagrangian eq. (22) we get the following reduced branching fractions:

$$\widetilde{Br}\big(J/\psi\to E_{\pi^+}\rho^-\big)=\widetilde{Br}\big(J/\psi\to E_{\pi^-}\rho^+\big)=\widetilde{Br}\big(J/\psi\to E_{\pi^0}\rho^0\big)=\left|g+\frac{1}{\sqrt{3}}g'_M+\frac{1}{3}g'_E\right|^2,$$
$$\widetilde{Br}(J/\psi\to E_{K^+}K^*-)=\widetilde{Br}(J/\psi\to E_{K^-}K^{*-})=\left|g-\frac{1}{2\sqrt{3}}g'_M+\frac{1}{3}g'_E\right|^2,$$
$$\widetilde{Br}\Big(J/\psi\to E_{K^0}\overline{K}^{*0}\Big)=\widetilde{Br}\big(J/\psi\to E_{\overline{K}^0}K^{*0}\big)=\left|g-\frac{1}{2\sqrt{3}}g'_M-\frac{2}{3}g'_E\right|^2,$$

$$\widetilde{Br}\big(J/\psi\to E_\eta\phi\big)=\left|g-\frac{2}{\sqrt{3}}g'_M-\frac{2}{3}g'_E\right|^2\left(\sqrt{\frac{2}{3}}\cos\varphi_P+\frac{1}{\sqrt{3}}\sin\varphi_P\right)^2,$$
$$\widetilde{Br}\big(J/\psi\to E_\eta\omega\big)=\left|g+\frac{1}{\sqrt{3}}g'_M+\frac{1}{3}g'_E\right|^2\left(\sqrt{\frac{1}{3}}\cos\varphi_P-\sqrt{\frac{2}{3}}\sin\varphi_P\right)^2,$$
$$\widetilde{Br}(J/\psi\to E_\eta\rho^0)=|g'_E|^2\left(\sqrt{\frac{1}{3}}\cos\varphi_P-\sqrt{\frac{2}{3}}\sin\varphi_P\right)^2,$$
$$\widetilde{Br}\big(J/\psi\to E_{\eta'}\phi\big)=\left|g-\frac{2}{\sqrt{3}}g'_M-\frac{2}{3}g'_E\right|^2\left(\sqrt{\frac{1}{3}}\cos\varphi_P-\sqrt{\frac{2}{3}}\sin\varphi_P\right)^2,$$

$$\widetilde{Br}\big(J/\psi \to E_{\eta'}\omega\big) = \left|g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E\right|^2 \left(\sqrt{\frac{1}{3}}\sin\varphi_P + \sqrt{\frac{2}{3}}\cos\varphi_P\right)^2,$$
$$\widetilde{Br}(J/\psi \to E_{\eta'}\rho^0) = |g'_E|^2 \left(\frac{1}{\sqrt{3}}\sin\varphi_P + \sqrt{\frac{2}{3}}\cos\varphi_P\right)^2,$$
$$\widetilde{Br}(J/\psi \to E_{\pi^0}\phi) = 0,$$
$$\widetilde{Br}(J/\psi \to E_{\pi^0}\omega) = |g'_E|^2. \quad (23)$$

From the above-reduced branching fractions, the following relations can be obtained:

$$\frac{\widetilde{Br}\big(J/\psi \to E_{\eta}\omega\big)}{\widetilde{Br}\big(J/\psi \to E_{\pi^0}\rho^0\big)} = \frac{\widetilde{Br}\big(J/\psi \to E_{\eta}\rho^0\big)}{\widetilde{Br}\big(J/\psi \to E_{\pi^0}\omega\big)} = \left(\sqrt{\frac{1}{3}}\cos\varphi_P - \sqrt{\frac{2}{3}}\sin\varphi_P\right)^2,$$
$$\frac{\widetilde{Br}\big(J/\psi \to E_{\eta'}\omega\big)}{\widetilde{Br}\big(J/\psi \to E_{\pi^0}\rho^0\big)} = \frac{\widetilde{Br}\big(J/\psi \to E_{\eta'}\rho^0\big)}{\widetilde{Br}\big(J/\psi \to E_{\pi^0}\omega\big)} = \left(\sqrt{\frac{1}{3}}\sin\varphi_P + \sqrt{\frac{2}{3}}\cos\varphi_P\right)^2. \quad (24)$$

Both because $Br(J/\psi \to \omega X(1835))$ is heavily suppressed (please see the previous section of this work) and because there are not yet any experimental reports on it, we take $\widetilde{Br}(J/\psi \to E_{\eta'}\omega) = |g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2(\sqrt{\frac{1}{3}}\sin\varphi_P + \sqrt{\frac{2}{3}}\cos\varphi_P)^2 \approx 0$, this implies $|g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 \approx 0$ or $(\sqrt{\frac{1}{3}}\sin\varphi_P + \sqrt{\frac{2}{3}}\cos\varphi_P)^2 \approx 0$. If $|g + \frac{1}{\sqrt{3}}g'_M + \frac{1}{3}g'_E|^2 \approx 0$, we can see $\widetilde{Br}(J/\psi \to E_{\pi^+}\rho^-) = \widetilde{Br}(J/\psi \to E_{\pi^-}\rho^+) = \widetilde{Br}(J/\psi \to E_{\pi^0}\rho^0) \approx 0$. However, if $(\sqrt{\frac{1}{3}}\sin\varphi_P + \sqrt{\frac{2}{3}}\cos\varphi_P)^2 \approx 0$, it indicates $\widetilde{Br}(J/\psi \to E_{\eta'}\rho^0) \approx 0$, and we cannot observe $X(1835)$ in the process $J/\psi \to \rho^0 X(1835)$, with $X(1835) \to p\overline{p}$ or $X(1835) \to \eta'\pi^+\pi^-$. This conclusion is consistent with the result $Br(J/\psi \to \rho X(1835)) < Br(J/\psi \to \omega X(1835))$ which has been obtained in sect. 2.2.

3.3 $J/\psi \to E_V P$

Similarly to the above two cases, the effective Lagrangian responsible for the decay is

$$\mathcal{L}_{eff} = \varepsilon_{\mu\nu\alpha\beta}F^{\mu\nu}_{\Psi}\Bigg\{g''_8 F^{\alpha\beta}_{E_{V_a}}P_a + g''_1 F^{\alpha\beta}_{E_{\omega_1}}P_{\eta_1} + \left[g''_{M,88}d^{abc}F^{\alpha\beta}_{E_{V_a}}P_b M^c + \sqrt{\frac{2}{3}}g''_{M,81}F^{\alpha\beta}_{E_{V_a}}M^a P_{\eta_1} + \sqrt{\frac{2}{3}}g''_{M,18}F^{\alpha\beta}_{E_{\omega_1}}M^a P_a\right] + \left[g''_{E,88}d^{abc}F^{\alpha\beta}_{E_{V_a}}P_b E^c + \sqrt{\frac{2}{3}}g''_{E,81}F^{\alpha\beta}_{E_{V_a}}E^a P_{\eta_1} + \sqrt{\frac{2}{3}}g''_{E,18}F^{\alpha\beta}_{E_{\omega_1}}E^a P_a\right]\Bigg\} \quad (25)$$

under the nonet symmetry, the relations $g''_8 = g''_1 \equiv g''$, $g''_{M,88} = g''_{M,81} = g''_{M,18} \equiv g''_M$ and $g''_{E,88} = g''_{E,81} = g''_{E,18} \equiv g''_E$ hold. From eq. (19) and the above Lagrangian eq. (25) we can obtain the following reduced branching fractions:

$$\widetilde{Br}\big(J/\psi \to \pi^+ E_{\rho^-}\big) = \widetilde{Br}\big(J/\psi \to \pi^- E_{\rho^+}\big) = \widetilde{Br}\big(J/\psi \to \pi^0 E_{\rho^0}\big) = \left|g'' + \frac{1}{\sqrt{3}}g''_M + \frac{1}{3}g''_E\right|^2,$$
$$\widetilde{Br}\big(J/\psi \to K^+ E_{K^{*-}}\big) = \widetilde{Br}\big(J/\psi \to K^- E_{K^{*+}}\big) = \left|g'' - \frac{1}{2\sqrt{3}}g''_M + \frac{1}{3}g''_E\right|^2,$$
$$\widetilde{Br}\big(J/\psi \to K^0 E_{\overline{K}^{*0}}\big) = \widetilde{Br}\big(J/\psi \to \overline{K}^0 E_{K^{*0}}\big) = \left|g'' - \frac{1}{2\sqrt{3}}g''_M - \frac{2}{3}g''_E\right|^2,$$
$$\widetilde{Br}\big(J/\psi \to \eta E_{\phi}\big) = \Bigg|g''\cos\left(\theta_P - \varphi_V\right) - \left(g''_M + \frac{1}{\sqrt{3}}g''_E\right) \times \left[\frac{1}{\sqrt{3}}\cos\theta_P\cos\varphi_V + \sqrt{\frac{2}{3}}\sin(\theta_P + \varphi_V)\right]\Bigg|^2,$$
$$\widetilde{Br}\big(J/\psi \to \eta E_{\omega}\big) = \Bigg|g''\sin\left(\varphi_V - \theta_P\right) + \left(g''_M + \frac{1}{\sqrt{3}}g''_E\right) \times \left[-\frac{1}{\sqrt{3}}\cos\theta_P\sin\varphi_V + \sqrt{\frac{2}{3}}\cos\left(\theta_P + \varphi_V\right)\right]\Bigg|^2,$$
$$\widetilde{Br}\big(J/\psi \to \eta E_{\rho^0}\big) = |g''_E|^2 \left(\frac{1}{\sqrt{3}}\cos\theta_P - \sqrt{\frac{2}{3}}\sin\theta_P\right)^2,$$
$$\widetilde{Br}\big(J/\psi \to \eta' E_{\phi}\big) = \Bigg| - g''\sin\left(\varphi_V - \theta_P\right) + \left(g''_M + \frac{1}{\sqrt{3}}g''_E\right) \times \left[-\frac{1}{\sqrt{3}}\sin\theta_P\cos\varphi_V + \sqrt{\frac{2}{3}}\cos\left(\theta_P + \varphi_V\right)\right]\Bigg|^2$$
$$\widetilde{Br}\big(J/\psi \to \eta' E_{\omega}\big) = \Bigg|g''\cos\left(\varphi_V - \theta_P\right) + \left(g''_M + \frac{1}{\sqrt{3}}g''_E\right) \times \left[-\frac{1}{\sqrt{3}}\sin\theta_P\sin\varphi_V + \sqrt{\frac{2}{3}}\sin\left(\theta_P + \varphi_V\right)\right]\Bigg|^2,$$
$$\widetilde{Br}\big(J/\psi \to \eta' E_{\rho^0}\big) = |g''_E|^2 \left(\frac{1}{\sqrt{3}}\sin\theta_P + \sqrt{\frac{2}{3}}\cos\theta_P\right)^2,$$
$$\widetilde{Br}\big(J/\psi \to \pi^0 E_{\phi}\big) = |g''_E|^2 \left(\frac{1}{\sqrt{3}}\cos\varphi_V - \sqrt{\frac{2}{3}}\sin\varphi_V\right)^2,$$
$$\widetilde{Br}\big(J/\psi \to \pi^0 E_{\omega}\big) = \left|g''_E\right|^2 \left(\frac{1}{\sqrt{3}}\sin\varphi_V + \sqrt{\frac{2}{3}}\cos\varphi_V\right)^2, \quad (26)$$

here θ_P is the mixing angle of η and η' with $\theta_P \approx -16.9° \pm 1.7°$ [41], and we can find the relation

$$\frac{\widetilde{Br}\big(J/\psi \to \pi^0 E_{\phi}\big)}{\widetilde{Br}\big(J/\psi \to \pi^0 E_{\omega}\big)} = \left(\frac{\frac{1}{\sqrt{3}}\cos\varphi_V - \sqrt{\frac{2}{3}}\sin\varphi_V}{\frac{1}{\sqrt{3}}\sin\varphi_V + \sqrt{\frac{2}{3}}\cos\varphi_V}\right)^2 = \left(\frac{1 - \sqrt{2}\tan\varphi_V}{\tan\varphi_V + \sqrt{2}}\right)^2. \quad (27)$$

In summary, the other exotic states in the weight diagram are expected to be observed in the future, and these relations between the branching fractions can be served as a guide to the experimental search for these exotic states.

4 Conclusion and discussion

In conclusion, the processes $\Upsilon \to \gamma X(1835)$ and $J/\psi \to \omega X(1835)$ have been investigated. Considering the large coupling of $X(1835)$ with $p\overline{p}$ and $\eta'\pi^+\pi^-$, we propose that $X(1835)$ is a baryonium with sizable gluon content, and mainly belongs to a $SU(3)$ flavor singlet. In this scheme, we can finely understand the observation data both in the process $J/\psi \to \gamma X(1835)$, $X(1835) \to p\overline{p}$ [2] and in $J/\psi \to \gamma X(1835)$, $X(1835) \to \eta'\pi^+\pi^-$ [1]. We estimate that in the $\Upsilon(1S)$ radiative decay the product branching fraction $Br(\Upsilon(1S) \to \gamma X(1835))Br(X(1835) \to p\overline{p}) < 6.45 \times 10^{-7}$, which is compatible with the CLEO's experimental upper limit $Br(\Upsilon(1S) \to \gamma X(1835))Br(X(1835) \to p\overline{p}) < 5 \times 10^{-7}$ [4]. In the processes $J/\psi \to \gamma X(1835)$ and $\Upsilon(1S) \to \gamma X(1835)$, the gluon component plays an important role due to the $U_A(1)$ anomaly, thus, we find out that the drastic smallness of $Br(\Upsilon(1S) \to \gamma X(1835))$ is caused the special nature of $X(1835)$, and it does not contradict the experimental evidence of $X(1835)$ revealed in the process $J/\psi \to \gamma X(1835)$ by BES.

In our baryonium scheme of $X(1835)$, we found out also that $Br(J/\psi \to \omega X(1835)) = (2.00 \pm 0.35) \times 10^{-5}$, $8.00 \times 10^{-7} < Br(J/\psi \to \omega X(1835))Br(X(1835) \to p\overline{p}) < 2.80 \times 10^{-6}$, and $2.40 \times 10^{-6} < Br(J/\psi \to \omega X(1835))Br(X(1835) \to \eta'\pi^+\pi^-) < 8.40 \times 10^{-6}$. The production of $X(1835)$ in the process $J/\psi \to \omega X(1835)$ is heavily suppressed. We also point out that $Br(J/\psi \to \rho X(1835)) < Br(J/\psi \to \omega X(1835))$, so it is very difficult to observe $X(1835)$ in the process $J/\psi \to VX(1835)$ (here V is ω or ρ) with $X(1835) \to p\overline{p}$ or $X(1835) \to \eta'\pi^+\pi^-$, and the J/ψ radiative decay is the most suitable place for searching $X(1835)$. We address that the baryonic component dominates the decay $J/\psi \to VX(1835)$ with V being ω or ρ, since the $U_A(1)$ anomaly contributions are suppressed in these processes. The experimental check for the above results is expected.

Finally, we conjecture the existence of a baryonium nonet, which is supported in ref. [11] and ref. [8], and the nonet can be pseudoscalar (E_{P_i}) or vector (E_{V_i}). The $p\overline{p}$ enhancement $X(1835)$ is identified as $E_{\eta'}$, and the $p\overline{\Lambda}$ enhancement [38] can be $E_{K^{*+}}$ or E_{K^+}. We derive the reduced branching fractions of $J/\psi \to E_P P$, $J/\psi \to E_P V$ and $J/\psi \to E_V P$ in a model-independent way basing on the $SU(3)$ symmetry with symmetry breaking included. The relations between the branching fractions can be served as a guide to the experimental search for these exotic states.

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